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ABSTRACT

Eleven students out of 94 selected a hexagonal container as larger than a triangular container having the same surface area, although the capacity of the former was 50% greater than the latter. When presented with hexagons and triangles of the same perimeter but different areas, 40 students selected the hexagon as the larger area. When the hexagonal and triangular containers were reconsidered, 51 chose the hexagonal container, 45 of whom used analogy with the area problem. Similar results were obtained using another transfer problem. (MP)

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A STUDY OF THE INTERFERING EFFECTS OF SOME DECEPTIVE VISUAL AND MATHEMATICAL CLUES RELATED TO AREA AND CAPACITY AND THE ABILITY OF SEVENTH AND EIGHTH GRADE STUDENTS TO USE ANALOGY TO SOLVE CAPACITY PROBLEMS

By Walter Szetela

ABSTRACT

Ninety-four seventh and eighth grade students were presented with carefully constructed containers having the same surface areas but different capacities. By visual inspection only 11 students selected a hexagonal container as larger than a triangular container although the capacity of the hexagonal container was fifty per cent greater than the triangular container. When presented with hexagons and triangles of the same perimeter but different areas, 40 students selected the hexagon as the larger area using visual inspection. When the hexagonal and triangular containers were reconsidered, the number of students correctly selecting the hexagonal container increased from 11 to 51. Of these correct solutions, 45 were obtained by analogy with the area problem. On a transfer problem involving containers with square and rectangular cross section, both with same surface area, 51 students identified the square container as the larger. Forty of these solutions were obtained by analogy.

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A STUDY OF THE INTERFERING EFFECTS OF SOME DECEPTIVE VISUAL AND MATHEMATICAL CLUES RELATED TO AREA AND CAPACITY AND THE ABILITY OF SEVENTH AND EIGHTH GRADE STUDENTS TO USE ANALOGY TO SOLVE CAPACITY PROBLEMS

When students and adults make size comparisons between different shaped areas and different shaped containers, they may be misled by visual deceptions or by application of incorrect mathematical generalizations. For example in the two triangles shown below, when students are asked to select the triangle of larger area, they are more likely to select the

triangle on the right because it "looks larger." In fact the triangles have congruent bases and altitudes and therefore have equivalent areas. The longer sides of the triangle on the right are a deceptive visual clue, an irrelevant consideration as the area of the triangle depends on the sizes of the base and altitude.

An example of an incorrect mathematical generalization is that equal perimeters imply equal areas. The two rectangles below have equal perimeters and students may state that the area of the two rectangles are the same when in fact the square rectangle has the larger area.

Similar deceptions occur with capacity comparisons. This study was conducted to investigate such deceptive influences among seventh and eighth grade students, and also to investigate the ability of students to use the problem solving heuristic of analogy in the solution of two problems on comparison of capacities of two containers. In particular the study was made to try to answer the following questions:

1. To what extent are seventh and eighth grade students able to identify the larger of two containers of different shapes but which are made of the same amount of material?
   a) When they have knowledge that the amount of material is the same?
   b) When they do not have knowledge that the amount of material is the same?
2. To what extent are seventh and eighth grade students able to identify the polygon of larger area given two different polygons known to have equal perimeters?
3. To what extent are seventh and eighth grade students able to relate the solution of an area problem to the solution of a capacity problem? (Use of analogy in problem solving)
4. To what extent are seventh and eighth grade students able to transfer the use of analogy in one capacity problem to solve a related capacity problem?

SUBJECTS OF THE STUDY. The subjects of the study were two seventh grade classes in a Vancouver school and two eighth grade classes in a school in Richmond, British Columbia. The seventh grade classes were from an open area school with no special selection procedures used to form the two classes and were considered of equal abilities by their two teachers. These classes were kept intact for this study. The eighth grade classes differed in ability, one being in a slightly higher academic program than the other. Twelve students were randomly selected from each of these two classes for each of two presentations of materials and instruction by the investigator. Thus each eighth grade group had 24 students, 12 from each of two academic levels. In the seventh grade classes 20 students were in one group and 26 in the other.

MATERIALS Special materials were constructed for the study. Full class size sets of the following materials were made:

1. Hexagonal and triangular shaped prisms. These containers were

made from identical rectangular strips of tagboard 36 cm long by 15 cm wide. The capacity of the hexagonal container was one and one half times as large as the capacity of the triangular container.

2. Hexagonal and triangular shaped polygons. All polygons had perimeters of 36 cm. The area of the hexagon was one and one half times the area of the triangle.
3. Two different shaped rectangular prisms. These containers were made from identical rectangles 20 cm long by 12 cm wide. One container had a rectangular cross section 8 cm long by 2 cm wide. The other container had a square cross section 5 cm on each edge.
4. Two large posters. One of the posters was to help convince each experimental group of students that the hexagonal and triangular containers were made from exactly the same shape and amount of material. The other poster was to provide information that would enable students to analogize from an area problem to a capacity problem.

METHOD OF PRESENTING MATERIALS AND INSTRUCTION The method of presenting the materials and instruction to the subjects varied only in the initial familiarization of the materials with the students. All students in each of four presentations were given hexagonal and triangular containers placed before each of them before any verbal instructions were given. A metric ruler was also provided for each student. The experimental seventh and eighth grade groups were each shown a poster while the instructor demonstrated how the hexagonal and triangular containers had been made from exactly the same sized and shaped rectangular strips. The control groups were not shown the poster or given any information as to how the containers were constructed. Students were instructed to work only with their own set of materials and not to communicate with other students during the presentation.

The investigator provided instructions and directions during four normal class periods, one period for each of the seventh and eighth grade classes. The investigator was assisted by a graduate student who distributed and collected materials quietly and efficiently so that there was no difficulty in preserving the continuity of the presentation. As each set of materials was examined, measured, and considered by the students, the students were directed to answer eight questions printed on two sheets. For each question enough time was allowed so that every student was able to write his responses and to give reasons for certain responses. It was necessary to write the reasons for some responses so

that the investigator could determine the type of mathematical thinking which each student used, and in particular whether he was using analogy in each of two capacity comparison problems.

The materials and instruction were presented in the following sequence:

1. Comparison of Capacities by Visual Inspection and Measurement.

   Each student examined the hexagonal and triangular container in his possession. After sufficient time for examination, measurement, and thought, he was directed to indicate which container he believed to be the larger. An alternative choice was to state that the containers had the same capacity. Students were also directed to write a reason for their selection.

2. Comparison of Areas of Different Polygons With Equal Perimeters

   Students examined regular hexagons and triangles of equal perimeters. They were directed to measure the perimeters of each polygon. Care was taken to insure that every student obtained the correct perimeters of 36 cm for both polygons. They were asked to decide which of the two polygons had the greater area. An alternative choice was that the polygons had the same area.

3. Demonstration that the Hexagon Was Larger than the Triangle

   Students were given a sheet which showed a hexagon and triangle exactly the same size as the polygons in the hands of the students. At the same time a large poster was displayed to show that the hexagon had more area than the triangle. This was done by showing that it took six color contrasting small triangles to cover the hexagon but it took only four color contrasting triangles of the same size to cover the large triangle. Students were questioned until all agreed that the hexagon was indeed larger in area than the triangle.

4. Reexamination of Original Problem to Test for Use of Analogy

Students were next instructed to reexamine the triangular and hexagonal containers considered at the outset of instruction. They were asked to think again and make a choice of the larger container. They were also asked to provide the reason for the choice. The previous activity had provided a basis for the use of analogy to solve the problem.

5. Determination of Transfer of Use of Analogy to New Problem

Students were given a rectangular container with a cross section measuring 8 cm by 2 cm and a square container 5 cm on a side. Both containers were the same height. The perimeters of both cross sections were 20 cm. With no additional instructions, students were asked to select the container with greater capacity and to give a reason for the selection. Some additional questions were asked to determine if the students were consistent in their responses and also to determine to what extent students estimated one container's capacity over another. The triangular container had two thirds the capacity of the hexagonal container, and the rectangular container had 16/25 the capacity of the square container.

RESULTS It was hypothesized that students who were shown that the hexagonal and triangular containers were made from the same sized and shaped rectangles would be less likely to identify the container of greater capacity than the students who were not given this information and demonstration. The results indicate that regardless of the presence or absence of this information, students had little success in correctly identifying the larger container. When the problem was first presented, only 11 out of 94 students correctly identified the larger container. The best performance was made by the grade 8 group which had been shown that the containers were made from the same amount of material. Six out of 24 of these students correctly identified the hexagonal container as the larger. Eighty-three out of 94 students were unable to select the larger container correctly even though the larger container had one and one half times as much capacity as the smaller. The summary of responses among the four groups is given in Table 1.

TABLE 1

STUDENTS' SELECTIONS OF LARGER CONTAINER USING VISUAL COMPARISON

	Correct Choice (Hexagonal) Container	Incorrect Choices (Triangular) Container	(Same)
Grade 7N (NOT shown that containers were made from same sized rectangles)	2	3	21
Grade 7S (SHOWN that containers were made from same sized rectangles)	1	0	19
Grade 8N (NOT shown that containers were made from same sized rectangles)	2	1	21
Grade 8S (SHOWN that containers were made from same sized rectangles)	6	0	18
TOTALS	11	4	79

Students had more success identifying the larger of two polygons of different shapes but having the same perimeter. Given a regular hexagon and an equilateral triangle whose perimeters students measured as 36 cm, students were better able to tell by visual inspection or otherwise that the hexagon had greater area than the triangle. The hexagon was one and one half times as large in area as the triangle. The summary of responses among the four groups is given in Table 2.

TABLE 2

STUDENTS' SELECTIONS OF LARGER AREA POLYGON UNDER VISUAL COMPARISON

	Identified Correctly Larger Area (Hexagon)	Incorrect Choices Triangle	Same
Grade 7N	10	1	15
Grade 7S	8	0	12
Grade 8N	8	0	16
Grade 8S	14	0	10
TOTALS	40	1	53

After students were shown that the area of the hexagon was one and one one half times as large as the area of the triangle by demonstrating that it took six small equilateral triangles to cover the hexagon while taking only four small equilateral triangles to cover the triangle, they were presented with the original capacity problem a second time. The purpose was to determine if they could use the information in the area comparisons to make an analogy with the capacity problem instead of just using visual comparison. In all four groups the number of students correctly identifying the larger container in the reconsideration of the problem was considerably greater than the number making correct identifications initially. Table 3 shows the number of correct identifications in each group and the number of students who actually used analogy with the area problem to make their correct choices. Most of the students who correctly identified the larger container actually gave reasons which indicated that they had solved the problem by analogy with the area problem. Of the 51 correct identifications, 45 were obtained through analogy with the area problem.

TABLE 3

STUDENTS' SELECTION OF LARGER CONTAINER ON RECONSIDERATION OF PROBLEM AND NUMBER OF SUCCESSFUL STUDENTS USING ANALOGY

	Incorrect Choices		Correct Choice	Number Using
	Triangle	Same	Hexagon	Analogy
Group 7N	0	12	14	12
Group 7S	0	8	12	12
Group 8N	0	14	10	10
Group 8S	0	9	15	11
TOTALS	0	43	51	45

Students were presented with a different but similar capacity problem to determine if they could transfer the analogy with area to a different problem. The containers had rectangular cross sections of equal perimeters, one with an 8 cm by 2 cm rectangular cross section, the other with a square cross section 5 cm on each side. Thus the perimeter of each cross section was 20 cm. The capacity of the rectangular container was 64 per cent of the capacity of the square container. In all four groups a considerably greater number of students

correctly identified the square container as the larger in the new problem, than were able to identify the larger container in the original problem using the hexagonal and triangular containers. Differences between the number of students who solved the new analogy problem and the number of students who solved the previous analogy problems were small. The number of correct solutions from seventh graders increased from 26 to 28 in the transfer problem. Among eighth graders this number decreased from 25 to 23. Table 4 gives the number of students reporting their selections of the larger container in the transfer problem as well as the number of students who used analogy to solve the problem.

TABLE 4

STUDENTS' SELECTIONS OF LARGER CONTAINER ON NEW CAPACITY PROBLEM AND NUMBER OF SUCCESSFUL STUDENTS USING ANALOGY

	Incorrect Choices		Correct Choice	Number Using
	Rectangular Container	same Capacity	Square Container	Analogy
Group 7N	3	8	15	10
Group 7S	0	7	13	12
Group 8N	0	14	10	10
Group 8S	1	10	13	8
TOTALS	4	39	51	40

CONCLUSIONS AND IMPLICATIONS

The results of this study appear to support the contention that seventh and eighth grade students do have great difficulty in comparing capacities of two containers even when one container is as much as one and one half times as large as another. In this study students who were given additional information which was expected to lead them astray, did no worse than students who were not given the information. However, the investigator did not anticipate that the groups from whom the misleading information was withheld, would discover a new source of misleading information by manipulating the hexagonal and triangular containers. The hexagonal container was flexible and it could be collapsed and deformed. A large number of students deformed the hexagonal containers into triangular containers which looked exactly like the given

triangular containers. These students concluded that the capacities of the two containers were therefore equivalent, in effect making an incorrect Piaget type conservation of capacity generalization. Fifteen of the 26 students in the seventh grade group which was not given the additional misleading information stated that the capacities of the containers were the same because one could be deformed to take the shape of the other. In the comparable eighth grade group, 8 of the 24 students also stated that the capacities were the same because of deformation from one container into another. Although much has been said and studied about various kinds of conservation, evidently more attention needs to be given to those instances where certain transformations do not result in conservation. Students should be given experiences which will enable them to resist such notions as preservation of area under redistribution of perimeter or preservation of capacity under deformation of shape.

The implications of these results therefore suggest that more investigative experience should be provided for students in intermediate grades on the concepts of perimeter and area, especially with figures of less conventional shapes. Present textbooks tend to rush students into formulas before they understand such concepts as perimeter and area. In upper intermediate grades students should have more exploratory experiences with different shaped containers and surfaces. Such experiences should be informal and include interesting problem solving activities. The use of formulas to obtain areas and volumes should be delayed until students have had sufficient experiences to acquire better understanding of seemingly simple, yet complex concepts of area and capacity.

Although the results of this study indicate that students are not as weak in comparing areas as they are in comparing capacities, the weakness is still severe. Fifty-four of 94 students failed to identify the hexagon as having greater area than the triangle even though the hexagon had fifty per cent more area. The equal perimeters of the two polygons was a powerful misleading clue for most of the students who reported that the areas were the same.

A very important outcome of this study is that the results indicate many students in seventh and eighth grades are apparently able to use analogy in solving capacity problems and quite likely other problems. Only 11 students out of 94 in grades 7 and 8 were able to solve the original capacity problem, but later when they had the opportunity to use analogy to solve the problem,

51 students were able to solve the problem with 45 of them using analogy. Most of the students who used analogy were able to use analogy again in a similar but different capacity problem. Thus it appears that teachers can present students with problem solving situations in which analogy is a possible means of solution. Students should be made conscious of the fact that they have used analogy whenever it occurs so that the use of analogy as a general problem solving heuristic will be attempted in other problems.

In summary the implications of the study are as follows:

1. The concepts of perimeter and area are not sufficiently well developed in intermediate grades. Much more practice needs to be given towards the acquisition of these concepts so that students may be better equipped to solve problems using these concepts.

2. The concept of capacity is even less well developed than that of area and perimeter. Practice with capacity using various shapes and sizes of containers needs to be given in informal investigative, and problem solving situations with the use of formulas delayed until the concepts of surface and capacity are better understood.

3. Analogy is a very useful problem solving strategy and can be employed in solving capacity problems. The best way for students to acquire some skill in using analogy as a problem solving heuristic is to give students experience with problems in which analogy is appropriate and where students have been given the necessary concepts to enable them to use analogy.

This study should be continued with other groups of students and problems solving situations. The study was done with students at two different grade levels in two different communities. Therefore a comparison of performances between the two grades was not appropriate here but should be one of the aims in future studies.